

# Effect of Shrink-Fitting and Magnetostriction on Core Loss and Vibration of Permanent Magnet Motor

Hassan Ebrahimi<sup>1</sup>, Yanhui Gao<sup>1</sup>, Hiroshi Dozono<sup>1</sup>, Kazuhiro Muramatsu<sup>1</sup>,  
Takashi Okitsu<sup>2</sup>, and Daiki Matsuhashi<sup>2</sup>

<sup>1</sup>Department of Electrical and Electronic Engineering, Saga University, Saga 840-8502, Japan

<sup>2</sup>Core Technology Development Division, Meidensha Corporation, Tokyo 141-0032, Japan

A method of three-dimensional finite element magneto-mechanical analysis is developed to investigate the effect of magnetostriction (MS) and stress, including shrink-fit stress and stress due to electromagnetism and MS, on core loss and vibration of motors. Shrink-fit stress calculation is carried out using static structural analysis and equivalent thermal force calculated by a novel method using the thermal stress tensor. Three-dimensional structural dynamic analysis is carried out with time step-size small enough to capture the vibration characteristics of electrical machines due to the inverter switching frequency. The method is applied to the core loss and vibration analysis of a permanent magnet motor. Numerical results show that the stator core loss increases significantly due to the shrink-fit stress. The stress, however, reduces the vibration, especially higher harmonics, through MS.

*Index Terms*—Magnetostriction, motor, stress, vibration.

## I. INTRODUCTION

COMPRESSIVE stress can affect the performance of electrical devices by altering the magnetic property of materials [1]-[3]. It also strongly affects the magnetostriction (MS) which, in turn, causes deformation and stress. Shrink-fitting of the stator leaves large compressive stress [3], [4] in the back-iron which increases the loss as well as MS parameter of the core in the stressed regions. In the calculation of fitting stress, most authors limit the problem to two-dimensions (2-D) [3], [5]. Moreover, the effect of shrink-fit stress on the vibration through MS has not been reported.

In this paper, a methodology is developed to investigate the effect of MS and stress, including the shrink-fit stress and the stress due to electromagnetism and MS, on the core loss and vibration of motors.

## II. METHOD OF ANALYSIS

### A. Equation of Motion and Stress Calculation

The finite element equation of motion takes the following form:

$$M\ddot{\mathbf{u}} + C\dot{\mathbf{u}} + K\mathbf{u} = \mathbf{f}, \quad (1)$$

where  $M$ ,  $C$  and  $K$  are the mass, damping, and stiffness matrices, respectively,  $\mathbf{u}$  is the displacement, and  $\mathbf{f}$  is the nodal force. In our proposed method, the force due to shrink-fitting, electromagnetism, and MS is given by the following equation:

$$\mathbf{f}_i = -\sum_e \int_{V_e} T \nabla N_i dV, \quad (2)$$

where  $\mathbf{f}_i$  is the force at node  $i$ ,  $T$  is a source stress tensor which will be explained in the next sub-sections, and  $N_i$  is the shape function of the node. The elastic stress  $\sigma$  is linked to  $\mathbf{u}$  through the following equations:

$$\sigma = D\varepsilon - \sigma^{\text{ref}}, \quad \varepsilon = \frac{1}{2}[\nabla \mathbf{u} + (\nabla \mathbf{u})^T], \quad (3)$$

where  $D$  is the tensor of elasticity,  $\varepsilon$  is the strain, and  $\sigma^{\text{ref}}$  is a reference with respect to which the stress is stated.

### B. Shrink-Fitting

In the shrink-fitting process, the source of loading is the thermal contraction. The source and reference tensors, proposed by the authors, are as follows:

$$T_{ij} = T_{ij}^{\text{th}} = \frac{-\delta_r}{r_{\text{in}}} \frac{E}{(1-\nu_p)} \delta_{ij}, \quad \sigma^{\text{ref}} = -T^{\text{th}}, \quad (4)$$

where  $T^{\text{th}}$  is called the thermal stress tensor in this paper,  $\delta_r$  is the radial interference,  $r_{\text{in}}$  is the housing inner radius,  $E$  and  $\nu_p$  are Young's modulus and Poisson's ratio, respectively, and  $\delta_{ij}$  is the Kronecker delta. Equation (1) is solved with  $C=K=0$ .

### C. Electromagnetism

A 2-D nonlinear magnetic field analysis using  $A$ -method coupled with circuit equations is carried out. The eddy current in the core as well as in permanent magnet is neglected. Tensors  $T$  and  $\sigma^{\text{ref}}$  are given by the following equations [1]:

$$T_{ij} = T_{ij}^{\text{m}} = B_i H_j - \left( \int_0^H \mathbf{B} \cdot d\mathbf{H} \right) \delta_{ij}, \quad \sigma^{\text{ref}} = 0, \quad (5)$$

where  $T^{\text{m}}$  is the Maxwell stress tensor, and  $\mathbf{B}$  and  $\mathbf{H}$  are the magnetic flux density and field intensity, respectively [1].

### D. Magnetostriction

This case is similar to the shrink-fitting and we have

$$T_{ij} = T_{ij}^{\text{ms}} = \frac{-E\lambda}{(1+\nu_p)} \frac{1}{2B^2} (3B_i B_j - \delta_{ij} B^2), \quad \sigma^{\text{ref}} = -T^{\text{ms}}, \quad (6)$$

where  $T^{\text{ms}}$  is the MS stress tensor and  $\lambda = \lambda(\mathbf{B}, \sigma_{\text{eq}})$  is the MS parameter of the material. The scalar  $\sigma_{\text{eq}}$  is the effective stress given by the following equation [2]:

$$\sigma_{\text{eq}} = \frac{3}{2} \hat{\mathbf{b}}^T s \hat{\mathbf{b}}, \quad \hat{\mathbf{b}} = \frac{\mathbf{B}}{B}, \quad (7)$$

where  $s$  is the deviatoric part of the stress tensor given by the following equation [2]:

$$s_{ij} = \sigma_{ij} - \frac{1}{3} \text{tr}(\sigma) \delta_{ij}, \quad (8)$$

where  $\text{tr}(\sigma)$  stands for the trace of  $\sigma$  which, in 3-D, becomes

$$\text{tr}(\sigma) = \sigma_{11} + \sigma_{22} + \sigma_{33}.$$

### III. NUMERICAL RESULTS

The method is applied to the core loss and vibration analysis of a 1.5 kW inverted-fed interior permanent magnet (IPM) motor. Fig. 1-(a) shows the 2-D finite element model of the motor and the MS parameter of the core  $\lambda$  is shown in Fig. 1-(b). For the mechanical analysis the 3-D model shown in Fig. 1-(c) is used. The motor is inverter-driven with the switching frequency of 7.2 kHz and the base frequency of 100 Hz. For the shrink-fit stress analysis, a fine mesh of one-tooth segment was constructed to take account of the laminated structure of the stator, but in the vibration analysis (Fig. 1-(c)) the stator is homogenized based on the rules of mixtures. Table I shows the mechanical parameters.

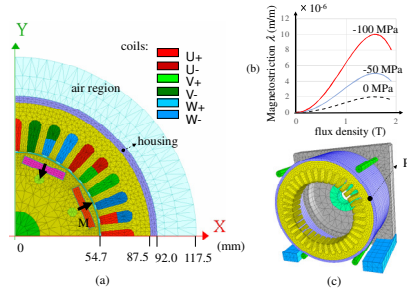


Fig. 1. (a) 2-D Finite element magnetic model of motor, (b) the MS parameter, (c) mechanical analysis model.

TABLE I  
MECHANICAL PARAMETERS OF THE MOTOR.

	$E$ (Pa)	$\nu_p$	Thickness (mm)	Mass density (kg/m <sup>3</sup> )
Laminate	$2 \times 10^{11}$	0.3	0.5	7800
Insulator	$1 \times 10^{10}$	0.3	0.02	1500
Others	$2 \times 10^{11}$	0.3	-	7800

#### A. Stress in Motor Core

According to the manufacture, radial interference of  $\delta_r=0.048$  mm was used in the thermal force calculation. The stress distributions due to shrink-fitting, electromagnetism, MS are shown in Fig. 2-(a) to 2-(f). It is obvious that the shrink-fit stress is well dominant, so in the core loss and vibration calculation, only the shrink-fit stress is considered.

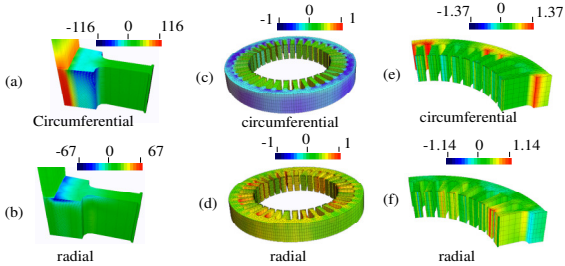


Fig. 2. Stress distributions in stator core; (a) and (b) due to shrink-fitting, (c) and (d) due to electromagnetism, and (e) and (f) due to MS. All stresses are in units of MPa with negative value for compressive stress.

#### B. Core Loss

Using the equivalent stress, given by (7), and the measurement data from shown in Fig. 3-(a) [4] and Fig. 3-(b) [5], the core loss of entire stator was calculated. The results, shown in Fig. 3-(c), indicates that the loss due to the shrink-fitting stress is nearly 0.7 percent of the motor nominal power.

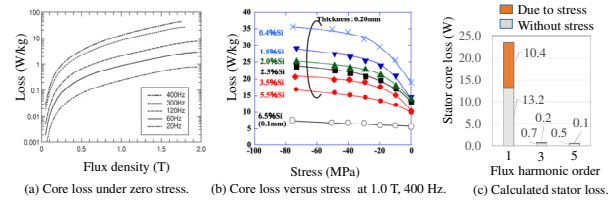


Fig. 3. (a) Core loss characteristics of the core (M43) under zero stress [4], (b) loss versus stress for various percentage of silicon [5], (c) comparison of calculated loss of the entire stator.

#### C. Vibration

The magnetic and MS forces were calculated for 1800 time steps in 2-D and after transferring the forces to the 3-D model, (1) with  $C=10^{-5}K$  was solved using the Newmark method [6]. Fig. 4-(a) compares the radial displacement of a selected point on the housing surface. The displacement increases significantly due to MS when the shrink-fit stress is considered, but as can be seen in Fig. 4-(b), the radial velocity which contributes to noise emission, even decreases especially at higher harmonics which are integers of the inverter switching frequency.

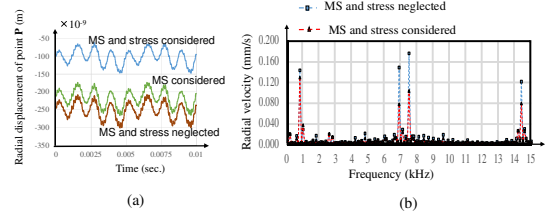


Fig. 4. (a) Comparison of radial displacement of a selected point on the housing, (b) Comparison of radial velocity spectra.

### IV. CONCLUSION

We developed a method to investigate the effect of magnetostriction and stress, including shrink-fit stress and the stress caused by electromagnetism and magnetostriction, on the core loss and vibration of motor. Numerical results reveal that fitting stress increases the core loss significantly but at the same time it causes a decrease in higher harmonics of the radial velocity, through dependence of MS on the stress.

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